Published by Institute of Physics Publishing for SISSA
Received: *April 10, 2005*
Accepted: *April 22, 2005*
Published: *May 6, 2005*

Brane necklaces and brane coils

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Abstract: We investigate the evolution of the networks of the cosmic strings in angled brane inflation. We show how they can be distinguished from the conventional ones. The cosmic strings in angled inflation are the daughter D_{p-2} branes that are extended between the mother D_p branes. In the effective action, the strings should have a moduli, since the endpoints of the D_{p-2} branes can move freely on the D_p branes. Then naturally the position of the D_{p-2} branes, which corresponds to the moduli of the $(1+1)$-dimensional effective action, can vary along the cosmic strings. The variation of the moduli results in the peculiar $(1+1)$-dimensional kink configurations. The kinks are the monopoles on the strings. Therefore, the cosmic strings in angled inflation become necklaces. The loops of the necklaces can shrink to produce stable winding states, which look like coils. We show why the cosmological implications of the brane necklaces are important. We point out that the cosmic strings in generic models of brane inflation should become necklaces, depending on the structure of the compactified space and the effective potential of the model.

Keywords: Solitons Monopoles and Instantons, Spontaneous Symmetry Breaking, D-branes, Cosmology of Theories beyond the SM.

Contents

1. Introduction

Models with more than four dimensions are interesting, because all the physical ingredients of the Universe can be unified in a higher dimensional theory. String theory is the most promising scenario where quantum gravity is included by the requirement of additional dimensions and supersymmetry. The idea of large extra dimension [1] is important, because it may solve the hierarchy problem. In this case, the observed Planck mass is obtained by the relation $M_p^2 = M_*^{n+2} V_n$, where M_* and V_n denote the fundamental scale of gravity and the volume of the n-dimensional compact space, respectively. In the scenarios of large extra dimension, the fields in the standard model are expected to be localized on wall-like structures ("our branes"), while the graviton propagates in the bulk. In the context of string theory, a natural embedding of this picture is realized by brane construction. The brane models are interesting from both phenomenological and cosmological viewpoints.

Analyses of cosmological defect formation are important in brane models.[1] For the cosmological scenarios, one can consider at least three different types of defects: 7

- Defects are branes.

  In this case, cosmological defects are the branes that have less than three spatial dimensions in the uncompactified space-time. In the previous discussions [11], in which the dynamical effect of "our brane" has been neglected, it was concluded that the cosmological production of monopoles and domain walls are negligible in the models of brane inflation. Later in ref. [9] and [10], however, it was shown that cosmological formation of such monopoles and domain walls are quite natural in generic cosmological scenarios.

[1]Inflation in models of low fundamental scale are interesting [2 – 4]. Scenarios of baryogenesis in such models are discussed in ref. [5 – 7], where defects play important roles. Brane defects such as monopoles, strings, domain walls and Q-balls [8, 9] are important because we are expecting that future cosmological observation will reveal the cosmological evolution of the Universe, which will also reveal the physics beyond the standard model. If one wants to know what kinds of brane defect are produced in the early Universe, one needs to understand how they are formed.

- Defects are deformed branes.

  In this case, the cosmological defects are formed by the continuous deformation of branes. First in ref. [12], and later in ref. [9, 13], it was shown that the fields that parameterize the position of branes can fluctuate in spatial directions to form cosmological brane defects. Cosmic strings are constructed in ref. [12], where the singularity in the core is resolved by smearing the wall-like structure. Monopoles, strings and domain walls are constructed in [9, 13].[2]

- Localized fields are shifted in the cosmological defects.

  In ref. [15], localization of matter fields on a fat domain wall is discussed to explain small interactions in the effective four-dimensional action. This idea is important in constructing realistic models in which proton must be stabilized. Along the line of this argument, new defect configurations are constructed in ref. [6], which induce shifts of the localization. Due to the shifts of the localization, small interactions can be enhanced in the core of these defects. Therefore, these defects can assist the generation of baryon number asymmetry of the Universe [6, 7].

First, we shall review the essence of the "previous arguments" [11], which were used to exclude the cosmological production of monopoles and domain walls. In the original scenario of brane inflation [16], inflationary expansion is driven by the potential between D-brane and anti D-brane evolving in the bulk. Then the scenario of the inflating branes at a fixed angle is studied in ref. [17], where the slow-roll condition is improved by introducing a small angle. The end of brane inflation is induced by the brane collision where brane annihilation (or recombination) proceeds through tachyon condensation [18]. During brane inflation, tachyon is trapped in the false vacuum, which can result in the formation of lower-dimensional branes after brane inflation. The production of cosmological brane defects was discussed in ref. [11], where it was concluded that cosmic strings are copiously produced in these scenarios, while monopoles and domain walls are negligible. To be precise, their arguments are based on the speculation that any variation of fields in the compactified direction must be suppressed during inflation since the compactification radius must be small compared to the horizon size during inflation. Then they have concluded that the daughter branes, which are formed by tachyon condensation on the world volume of the mother branes, must wrap the same compactified space as the mother branes. In this case the effect of compactification seems significant. As a result, it was concluded that the codimension of the daughter branes must lie within the uncompactified space. Since the number of the codimension must be even in this case, the cosmological defects must be cosmic strings. Moreover, later in ref. [19], it was discussed that the analysis does not fully account for the effects of compactification, because the directions transverse to the mother branes had not been considered. The effect of the RR fields extended to the compactified dimensions was discussed in ref. [19]. Then it was concluded that the creation of the gradients of the RR fields in the bulk of the compactified space is costly in energy, so

[2]It is possible to construct branes as defects in higher dimensional gauge theories. Interesting applications of this idea are discussed in ref. [14].

that there should be a serious suppression in the creation of the daughter D brane as far as they do not fill all the compactified dimensions. As a result, in the "previous arguments", cosmological brane defects were discussed to suffer from many unnatural constraints.

However, here we should recall that no such suppression has been discussed so far in the cosmological production of the conventional defects in four-dimensional effective gauge theory, while the realistic brane models must reproduce the standard model in its effective action. Therefore, it is quite natural to think about the following questions. Is it really impossible to produce branes that do not wrap the same compactified space as the mother branes? Is it really impossible to produce monopoles and domain walls in brane inflation? Here we should note that the "previous arguments" are not fully reliable in more realistic cases. The most obvious example was first discussed in ref. [20],[3] where the inconsistency of the string tension is solved in angled inflation. In the original argument in ref. [11], the tension of the string did not coincide with the one calculated from the effective action. This inconsistency is clearly due to the assumption that the daughter brane must wrap the same compactified space as the mother brane. In fact the previous arguments might be true in some simplest cases, however we must be more careful about the process of the brane recombination. In ref. [9, 20], it was shown that the production of the cosmic strings in angled inflation is realized by the creation of the daughter branes that are *extended* between the splitting mother branes. It should be noted that the creation of such extended branes is not a counter example to the generic mechanism of tachyon condensation. To be precise, one can understand from a careful treatment of the effective action [21] that the eigenfunction of the tachyonic mode should be localized at the intersection. Since the mechanism of this localization is *different* from the Kibble mechanism, this argument does not contradict to the "previous arguments". Therefore, the "seed" must be localized at the intersection. As the recombination proceeds, the D_{p-2} brane at the intersection is pulled out of the mother D_p branes, and finally becomes extended between mother branes. In this case the serious problem of the RR field is also avoided, since the length of the extended daughter brane vanishes when it is created at the intersection. It costs energy to pull D_{p-2} branes out of the D_p branes, however the cost is paid by the repulsive force between the splitting D_p branes. Of course the tension of such extended daughter branes matches precisely to the D-term strings in the effective action. Moreover, this conclusion is consistent with the analysis of the string production in the effective action, where the production of the D-term strings is not suppressed. Of course, it seems obvious from the string perspective that the brane creation is due to the local dynamics of the open strings at the intersection of the splitting D_p branes, which suggests the obvious localization of the eigenfunction. The daughter branes satisfies the required properties. Thus at least in the models of angled inflation, it is obvious that the cosmic strings in the effective action do not wrap the same compactified space as the mother branes.

From the above discussions, it is natural to think that the previous arguments are not reliable and should be compensated in more generic cosmological scenarios. Obviously,

[3] See also ref. [9].

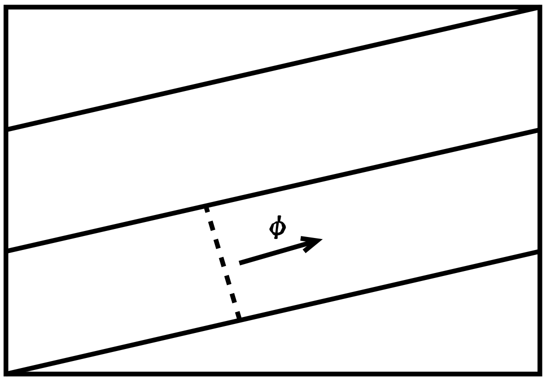

Figure 1: There is a direction in the compactified space along which the branes(=strings) can move. The flat direction is denoted by ϕ.

there is no reason that we must believe that in their final state the daughter branes must wrap the same compactified space as the mother branes. Since the previous argument cannot fully account for the generic process of the cosmological defect formation, one can hardly accept their conclusions. Is it really impossible to produce sufficient amount of cosmic monopoles and domain walls by the brane creation? This question is answered in ref. [9, 20], paying careful attention to the brane dynamics after inflation, especially to the dynamics of "our brane". For the extended branes to be produced between mother branes, the distance between mother branes is required to be zero at a moment. This idea is generic and applicable to other conventional cosmological processes. The spontaneous symmetry breaking in the effective action is sometimes described by the recombination of the branes [22] or by the branes falling apart, which can be induced by the thermal effects [23]. As a result, contrary to the previous speculation, it was shown [9] that there is no reason that one should believe that monopoles and domain walls [24] are suppressed in the brane Universe, once the dynamics of "our brane" is included.

Besides the cosmological defects that are formed by the brane creation, one can consider the defects that are formed by the continuous deformation of the branes. The two kinds of the brane defects can be produced by the same process [9, 13]. Therefore, the actual cosmological defects should be *the mixture of these defects.* It should be noted that the analyses of such mixed defects are quite important in understanding the evolution of the brane Universe.

From the above viewpoints, we shall reconsider the evolution of the cosmic strings in angled inflation. In this paper, we show that the strings in angled inflation can be distinguished from the conventional cosmic strings. In angled inflation, cosmic strings are the daughter D_{p-2} branes that are extended between mother D_p branes. The D_{p-2} branes have the flat direction (moduli) in the compactified space, along which the endpoints of the D_{p-2} branes can move freely on the D_p branes.[4] Then the position of the D_{p-2} branes can vary along the cosmic strings, which results in the peculiar type of $(1+1)$-dimensional kink configurations. One can also find the peculiar winding states that are defined on

[4]See figure 1.

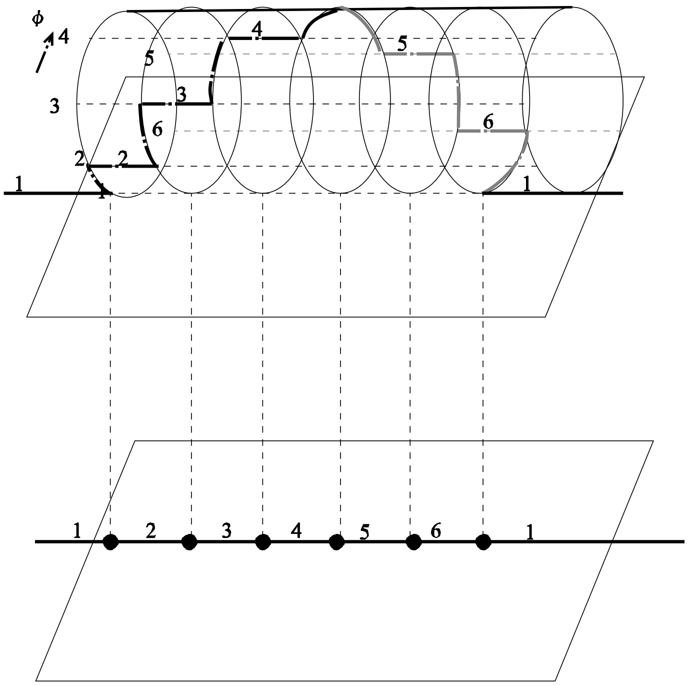

Figure 2: Strings in angled inflation could have many domains. The domains are interpolated by the kinks on the strings. Since the kinks are the monopoles skewered with the strings, the strings become brane necklaces.

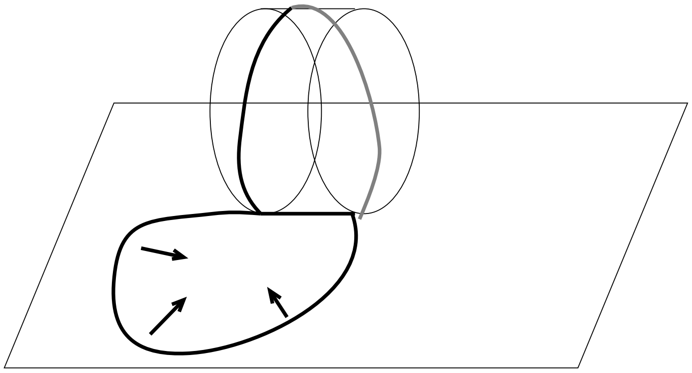

Figure 3: A chopped loop of the brane necklace can shrink. However, it cannot shrink to a point if the loop winds around compactified space.

the world volume of the strings. A schematic picture of the kinks is shown in figure 2. The point-like objects that appear on strings are monopoles. Since the monopoles are skewered with the strings, the cosmic strings in angled inflation become necklaces. These kinks are produced by the spatial deformation of the D_{p-2} branes. Therefore, the brane necklaces are the *hybrid* of the brane creation and the brane deformation. The chopped loops of the necklaces can shrink to produce the stable winding states, which looks like coils. A schematic picture of the brane coils is shown in figure 3. It should be noted here that the brane coils wind around the compactified space that is *different* from the mother branes. Of course the mechanism that induces such windings is different from the

Kibble mechanism. Therefore, our mechanism avoids the serious constraints that have been discussed previously in ref. [11].

2. Brane necklaces

In the early Universe there could be a pair of brane anti-brane separated in the compactified space. The potential energy of these branes can drive brane inflation, then they annihilate rapidly at the end. If this process contains only a pair of $D_p\overline{D}_p$ branes, it might be true that only cosmic strings are produced after inflation.[5] In this case, the cosmic strings are the daughter branes that wrap the same compactified space as the mother branes. At first the strings could move freely in the compactified space, but later their position should be fixed as was discussed in ref. [25, 26]. Then, if one assumes that other branes (including "our branes") are decoupled, these strings cannot break nor dissolve into other branes. In this case, since other branes are completely decoupled from the cosmic strings, the interactions of the cosmic strings must be quite simplified. The cosmological consequence of this scenario is already investigated in ref. [25, 26].

On the other hand, in generic cases it seems rather difficult to obtain enough number of "decoupled" cosmic strings while reheating the Universe to an acceptable temperature. If the moving brane of the brane inflation collides to the stack of the Standard Model (SM) branes, reheating should be successful. However, in this case one cannot ignore the interaction between the daughter branes and the other branes at the stack. Since the cosmic strings are the lower-dimensional branes that could break or dissolve into other branes, in generic cases one cannot simply ignore the interaction between other branes. Therefore, it is important to investigate cosmic strings that intersect with other branes.

From the above viewpoints, we shall try to understand the evolution of the cosmic strings in angled inflation to show how one can distinguish the cosmic strings in angled inflation from the conventional ones. As we have discussed in the previous section, angled inflation is the most obvious generalization of the simplest brane inflation, where the slow-roll condition is improved and the *cosmic strings intersect with "our branes"*. If the potential that lifts the flat direction of the cosmic strings is shallow, the Brownian motion in the compactified space must be important. The Brownian motion in the direction that is perpendicular to the daughter brane can be induced by the brane collision [27]. Even if the potential is not shallow due to the stringy effects, it seems natural to expect that the internal position of the cosmic strings can vary because of the energetic process of the brane collision. In any case, it seems too optimistic to expect that the potential is so steep that one can avoid the generation of the kinks on the strings. Moreover, if the angle is small($\theta \ll 1$) and the final state of the recombined mother brane wraps about θ^{-1} times around the compactified space,[6] the potential should have many local minima. These minima appear as the "domains" on the strings. To be precise, strings can have several distinctive regions(domains) where the branes are located at different sites in the

[5] Recently, it was shown by numerical simulations that many kinds of branes may appear in the intermediate state, which may annihilate to produce stable monopoles and domain walls in their final state [10].

[6] See figure 4.

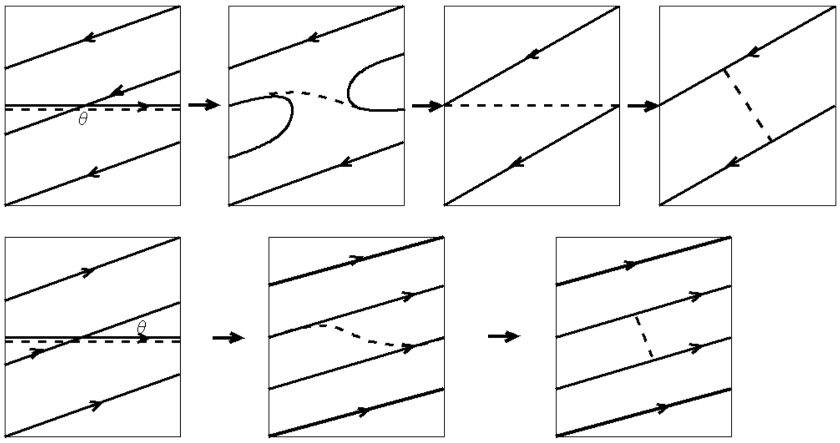

Figure 4: Upper row: schematic recombination of two D_p-branes with $(\pi-\theta) \ll 1$. The dashed line on the D_p-brane represents the D_{p-2}-brane that might appear on the worldvolume of the D_p-brane when the tachyon condenses. As the recombination proceeds, the D_{p-2} brane is pulled out from the mother brane, and finally becomes extended between the mother brane. Second row: schematic recombination of two D_p-branes with $\theta \ll 1$. In both cases the daughter D_{p-2} brane does not wrap the same compactified space as the mother brane.

compactified space.[7] Then kinks (i.e. (1+1)-dimensional domain walls) will appear on the strings, which interpolates between neighboring domains on the strings. These kinks may have interesting consequences.[8] Moreover, it is interesting to think about the configuration where the internal position of the cosmic string is changed so that it finally wraps around a non-trivial circle of the compactified space. Then a peculiar type of topological number can be defined *on* the strings, which can stabilize the chopped loops.[9] The winding states of the strings look like coils. Even though the brane coils are stabilized by their windings, they can be annihilated with another coils that have the opposite windings.[10]

The cosmological evolution of the conventional necklaces is already investigated in ref. [28].[11] Therefore, it is quite interesting to investigate the brane necklaces so that we can find the crucial difference from the conventional necklaces. Because of their peculiarity, the brane necklaces can be used to distinguish the brane Universe from the conventional one. Of course the above speculation seems rather naive, therefore we shall discuss this issue in more detail.

[7] See figure 2.

[8] Here we should note that the "kinks" on the strings are different from the "kinks and cusps" that are sometimes used in the discussion about the decay of the conventional cosmic strings.

[9] See figure 3.

[10] As is already discussed in ref. [12, 13], branes might be delocalized (smeared) in the core of a defect. If such delocalization is energetically favored, the loops around compactified space could be unwinded. In our model, however, it is obvious that the process is corresponding to the instanton-like phase transition, which is suppressed by the significant exponential factor.

[11] BPS necklaces in SQCD are discussed in ref. [14, 29].

2.1 String production after angled brane inflation

To make our discussions simple and convincing, here we consider angled brane inflation with a small angle($\theta \ll 1$). In this case, the eigenfunction of the tachyonic mode should be localized at the intersection. It should be noted that this mechanism of the localization is *different* from the conventional Kibble mechanism, which suggests that our result does not contradict to the previous arguments. Moreover, comparing the effective tension of the daughter branes with the one obtained in the effective action, one can easily confirm that the cosmic strings in angled inflation must be extended between mother branes.[12] If the daughter branes are extended between the mother branes as is shown in figure 1, it is easy to see that the daughter branes can move freely along the mother branes. The flat direction that corresponds to the free motion in the compactified space is depicted in figure 1. Of course, we know that the flat direction is supposed to be lifted by the potential that is due to the conventional supersymmetry breaking, which induces the mass of $O(m_{3/2})$. On the other hand, since we are considering small angle $\theta \ll 1$, the flat direction ϕ is periodic and winds about $\sim \theta^{-1}$ times around the compactified space. Therefore, the number of (local or global) minima should be as large as $\theta^{-1} \gg 1$. If the potential that lifts the flat direction is shallow, the brane strings can initially move freely along the flat direction. Even if the potential is not so shallow due to the stringy effects, one can naturally expect that the collision is so energetic that the daughter branes can stay at any minima. Branes located at different sites in the compactified space cannot intersect with each other [25, 26], because their separation in the compactified space is larger than the string scale.

Let us consider the $(1+1)$ dimensional world on the strings. The $(1+1)$ dimensional world on the strings will have many domains of the number of $\sim \theta^{-1}$, which correspond to the degenerated or quasi-degenerated vacua on the flat direction. Then the domain walls will appear on the strings, which interpolate between the two neighboring domains.[13] The width of the domain walls is determined by the shape of the effective potential $V(\phi)$, which becomes $\delta_m \sim m_{3/2}^{-1}$ if the potential is induced by the conventional mechanism of supersymmetry breaking, or can become as thin as $\delta_m \sim M_*^{-1}$ if the potential is lifted by the stringy dynamics.

- In the former case, the height of the effective potential is $V_0 \simeq m_{3/2}\Delta\phi$, where $\Delta\phi \simeq R_E M_*^2$ denotes the typical distance between the neighboring vacua. From the dimensional argument one can easily understand that the mass of the monopoles is $m \sim \Delta\phi$.[14]

[12] See figure 4.

[13] See figure 2.

[14] It should be noted that the effective action of the $(1+1)$ dimensional world of the strings must have non-zero cosmological constant due to the tension of the string in the background. Therefore, the precise formula of the effective potential should contain the effective cosmological constant, which may or may not dominate the vacuum energy. Since the cosmological constant in this case is nothing but the tension of the strings, one can easily understand that kinks are point-like objects that are "skewered" with the strings. On the other hand, if one considers point-like objects to which two strings are "attached", the definition of the mass should be obscured because of the contribution from the string tension. In this paper, we shall consider the former (skewered) picture for fat monopoles. See also figure 5.

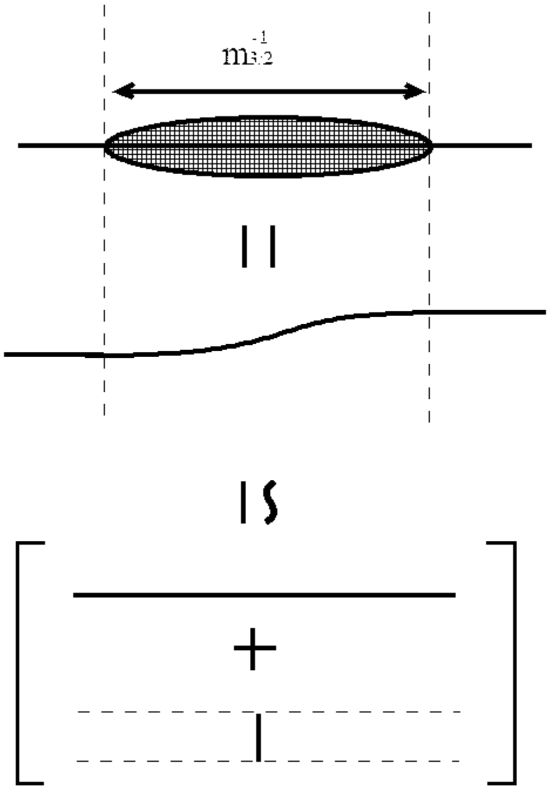

Figure 5: A fat monopole is skewered with a string.

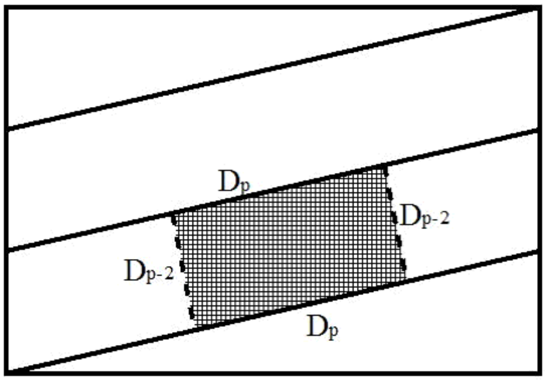

Figure 6: A D_{p-2} brane is expanded in the (p-2)-dimensional compactified space. The boundary of the D_{p-2} brane is D_p and D_{p-2} branes.

- From the brane perspective the kinks are corresponding to the D_{p-2} branes that are extended along the internal direction.[15] The kinks are nothing but the monopoles, which correspond to the D_{p-2} branes that are extended in the $(p-2)$-dimensional compactified space. Then one can easily calculate the effective mass of this brane configuration, which becomes $m \sim M_*^3 R_E^2 \theta$.

[15]See figure 6.

As we have discussed above, there are two different contributions to the mass of the monopoles. When one considers the SQCD-MQCD correspondence, one should think that the potential in the effective action must properly represent the brane dynamics. Of course, in such cases the two must agree [29]. On the other hand, if the effective potential that lifts the flat direction is due to the undefined mechanism of supersymmetry breaking, the results cannot agree. Therefore, in our case the mass of the monopoles becomes $m \sim \Delta\phi$ in the limit $\theta \to 0$, while the mass is dominated by the brane tension if $1 \gg \theta > (M_* R_E)^{-1}$. In this paper we shall consider these two different limits without specifying the origin of the effective potential.

2.2 Stability and decay of brane necklaces

Let us consider the evolution of the network of the brane necklaces. The strings at different domains cannot reconnect because they are placed at a distance in the compactified space. Therefore, in the length scale that is much larger than the distance between the monopoles, the reconnection probability of the necklaces is suppressed by the number of the minima:

$$p \simeq \theta \, . \tag{2.1}$$

The evolution of such strings is already discussed in ref. [26]. According to ref. [26], one can easily understand that the cosmic strings in $\theta \ll 1$ angled inflation are quite important in the observation of the gravitational wave signals. However, here we must be cautious because the evolution of the network of the necklaces can be different from the conventional $p \ll 1$ strings. Therefore, we should first examine the effect of the monopoles that might or might not affect the evolution of the network of the $p \ll 1$ strings. Of course, it is apparent that the details must be investigated using numerical simulations, however here we can modestly assume that one can use the conventional analyses as far as the parameter space of the model is within the range of ref. [26, 28].

To make our arguments clear, we use the same notations as in ref. [26, 28]. In ref. [28], it was assumed that the monopoles are formed *before* the strings are produced at the second phase transition. In our case, however, the monopoles are formed *after* the strings are formed. Since the succeeding evolution of the necklaces is irrelevant to how they are formed, the analysis in ref. [28] is applicable in our case. The sequence of the phase transition is only relevant to their initial condition. For example, if monopoles are produced long after strings are produced, their initial average separation d will be large compared to the original analysis of the conventional necklaces.

- If the potential is lifted by the conventional mechanism of supersymmetry breaking, the temperature T_m when monopoles are formed can be estimated as $T_m \sim (m_{3/2} M_p)^{1/2}$. In this case, the width of the monopoles δ_m is as large as $\delta_m \sim m_{3/2}^{-1}$. Therefore, their initial average separation d is inevitability as large as the horizon size.

- As we have stated above, one can consider steep potential that could be induced by the stringy effects. In this case, monopoles should be produced at the same time when strings are produced, which suggests that the initial average separation between monopoles can range from the minimal distance $d \sim M_*^{-1}$ [10] to the horizon size.

Here we introduce an important quantity for the evolution of the necklaces, which is the dimensionless ratio $r = m/(\mu d)$. Here μ and d denote the tension of the strings and the separation between the monopoles, respectively. In general, the initial value of r can be large or small, depending on the nature of the two phase transitions. As we have discussed above, we are considering two different limits.

- When monopoles are thin and are dominated by the brane dynamics, the mass of the monopoles is $m \simeq M_*^3 R_E^2 \theta$. In this case we can assume $d > M_*^{-1}$, which results in the initial value of the ratio $r_{ini} < M_* R_E$.
- On the other hand, if the potential is almost flat and is lifted by the conventional mechanism of supersymmetry breaking, the mass of the monopoles becomes $m \simeq \Delta\phi$. In this case, since the width of the monopoles is as large as $m_{3/2}^{-1}$, we must assume $d > m_{3/2}^{-1}$. Therefore, the initial value of the ratio is $r_{ini} < m_{3/2}/(M_* \theta)$.

In both cases we can put a modest assumption that $r_{ini} < 1$. Following the arguments in ref. [28], we can obtain the following equation for $r(t)$:

$$\frac{\dot{r}}{r} = -\frac{\kappa_s}{t} + \frac{\kappa_g}{t}, \tag{2.2}$$

where the first term on the right hand side describes the string stretching due to expansion of the Universe, while the second term describes the competing effect of string shrinking due to gravitational radiation. In this regime, one can find that if r is initially small it will grow at least until $r \sim 1$. As r grows, monopole anti-monopole (or coil anti-coil) annihilation should become important and the growth of r will terminate.

- If the potential is steep and the mass of the monopoles is given by $m \simeq M_*^3 R_E^2 \theta$, the width of the strings δ_s should be comparable to the width of the monopoles δ_m. Therefore, in this case one can assume that $\delta_s \sim \delta_m \sim M_*^{-1}$, which leads to the result $r_{\max} \simeq R_E M_*$. This result suggests that as far as $R_E M_* < 10^6$ the parameter space of our model is within the range of the original argument in ref. [28]. If $r \gg 1$, the loop self-intersections should be frequent and their fragmentation into smaller loops is very efficient. Therefore, a loop of size l typically disintegrates on a time scale
$$\tau_{r \gg 1} \sim \frac{l}{\sqrt{r}}, \tag{2.3}$$
which modifies the conventional analysis of the $p \ll 1$ strings.
- If the potential is almost flat and the mass of the monopoles is given by $m \simeq \Delta\phi$, the maximum value of the ratio becomes $r_{\max} \simeq m_{3/2}/(M_* \theta)^{-1} \ll 1$, which suggests that the monopoles are negligible. In this case, the standard evolution of the string network is applicable.

Let us recall the main quantities that are used in the standard analysis of the evolution of a string network. The long string network is characterized by the parameters $\xi(t)$, $L(t)$, and $l_{\rm wiggles}(t)$. Here $\xi(t)$ is the coherence length, which is defined as the distance beyond which the directions along the string are uncorrelated. $L(t)$ is the average distance between the strings, and $l_{\rm wiggles}(t)$ is the characteristic wavelength of the smallest wiggles that appear on long strings. The length $l(t)$ of the chopped-off loops are characterized by the parameter α,

$$l(t) = \alpha t \,, \tag{2.4}$$

where the standard value of α is determined by the gravitational radiation losses from loops, which is given by

$$\alpha^{st} \sim \Gamma G\mu \,, \tag{2.5}$$

where Γ is a numeriacl coefficiend of O(10). If the gravitational radiation from counter-streaming wiggles on long strings is much less efficient in damping the wiggles than ordinarily thought, α should be much smaller than α^{st} [30]. In our case, if the ratio r is small, the equation of motion is not different from the original argument, which means that we can examine the evolution of the network just using the results obtained in ref. [26]. In the macroscopic scale where the reconnection probability is small, one can estimate the above parameters as $L(t) \sim p^{1/2}t$ and $\xi(t) \sim t$. A significant fraction of the total string length within Hubble volume should go into loops each Hubble time, whose number density is

$$n(t) \sim \frac{1}{p\Gamma G\mu t^3} \,, \tag{2.6}$$

which is precisely the same as the original result.

However, what we should show in this paper is the difference from the standard results, which can be used to distinguish the brane necklaces from the conventional defects. In the original argument in ref. [26], the reconnection probability of the strings is $p \ll 1$ at any time during the evolution of the network. However, in our case the small reconnection probability is due to the fact that the strings in different vacua cannot reconnect. Therefore, in the macroscopic scale which is much larger than the distance d, the brane necklaces look precisely the same as the original $p \ll 1$ strings, while in the microscopic scale they look quite different. For example, if the wavelength of the wiggles $l_{\rm wiggles}$ is much smaller than d, the production of the small loops is efficient and at least in the local region the brane necklaces look like conventional $p \sim 1$ strings. On the other hand, if the wavelength of the wiggles $l_{\rm wiggles}$ is much larger than d, one can easily understand that the evolution of the brane necklaces is quite similar to the $p \ll 1$ strings. Therefore, in the case of $r \ll 1$, we can conclude that the evolution of the brane necklaces is precisely the same as the conventional $p \ll 1$ strings as far as

$$l_{\rm wiggles}(t) \gg d(t) \tag{2.7}$$

is satisfied. The above relation (2.7) puts a cut-off time for the original analysis of the $p \ll 1$ strings, which is not important for the GW burst analysis at the present cosmic time. The numerical simulations of this kind are difficult, because of the two hierarchically different scales. However, what is important for the observation of the GW emitted from strings is the behaviour at the macroscopic scale. Therefore, it must be important to note that at later period the behaviour of the brane necklaces is quite the same as the conventional $p \ll 1$ strings, which is already discussed in ref. [26]. Thus we can conclude that the brane necklaces can have significant impact on the observation of the gravitational wave. Please be sure that in our model the small reconnection probability is due to the small angle.

Our next example is the peculiar case of $r \gg 1$. In this case, the characteristic length scale of the network will be modified [28] as:

$$\xi(t) \sim (r+1)^{-1/2} t \,. \tag{2.8}$$

The string length per unit volume is $\sim p^{-1}\xi^{-2}$. The number of loops formed per Hubble volume per Hubble time is therefore [26]

$$N_l \sim \frac{r}{p\alpha} \,. \tag{2.9}$$

The loop self-intersections should be frequent and their fragmentation into smaller loops is very efficient, which suggests that the lifetime of a loop of size l is $\tau_{r\gg 1} \sim l r^{-1/2}$ [28]. In this case, it is easy to see that the loops produced by the necklaces of $r \gg 1$ are short-lived and the calculation of the loop density is similar to the cases of small α. The corresponding loop density is therefore

$$n(t) \sim \frac{r^{1/2}}{pt^3} \,. \tag{2.10}$$

Here we have assumed that the wiggles are small compared to the typical scale of the network, so that $l_{\text{wiggles}} < \xi$. The results obtained here suggests that the analysis of the GW signals emitted by the network of the cosmic necklaces are similar to the original argument [26]. The difference that we have found is the suppression factor $\Gamma G\mu r^{1/2}$ in their loop density.

Therefore, we may conclude that there is no qualitative difference between the brane necklaces and the conventional strings(or necklaces) as far as the observation of the gravitational wave signals is concerned. However, as we have discussed above, chopped loops of the brane necklaces can be stabilized due to their non-zero windings around compactified space. The stable relics of such loops are superheavy, which must satisfy the cosmological bound of the conventional cold dark matter. Therefore, in order to find qualitative differences, it is quite important to calculate the relic density of the brane coils.

To calculate the relic density of the stable relics, it is convenient to introduce the relative abundance $Y_s = n(t)/s(t)$, where $s(t)$ is the entropy density. In our case, small loops are produced through multiple self-interactions of larger loops, whose sizes are determined by the wavelength of the wiggles $l_{\text{wiggles}} \sim \alpha t$. The conventional analysis of $p \ll 1$ string network is reliable as far as $l_{\text{wiggles}} \gg d$.

- Let us first consider the case where the effective potential $V(\phi)$ is due to the stringy dynamics. In this case, monopoles are produced by the phase transition just after angled inflation. The number of the loops formed per Hubble volume per Hubble time is already given in ref. [26]. For $r < 1$, it becomes

$$N_l \sim \frac{1}{(p\alpha)}, \tag{2.11}$$

which cooresponds to the number of the coils emitted from the network of the brane necklaces. Therefore, we can write

$$\dot{n}_{coil} \sim \frac{1}{(p\alpha t^4)}. \tag{2.12}$$

In the most efficient case when all the loops are stabilized by the coils, the relic number density of the coils is obtained by integrating eq. (2.12). The ratio $Y_s = \int (\dot{n}/s) dt$ is:

$$Y_s \simeq (p\alpha)^{-1} M_p^{-3/2} \epsilon_t^{1/2} t_m^{-3/2} \tag{2.13}$$

for $r < 1$, and

$$Y_s \simeq \left(\frac{r}{p\alpha}\right) M_p^{-3/2} \epsilon_t^{1/2} t_m^{-3/2} \tag{2.14}$$

for $r \gg 1$. Here t_m is the time when the monopoles are produced as the kinks on the strings, and ϵ_t is defined by $\epsilon_t = t_m/t_{coil}$, where t_{coil} is the cut-off time when the wavelength of the wiggles becomes much larger than the typical distance between monopoles. We have assumed that the coils produced before t_{coil} are negligible.

- We must be careful if the effective potential $V(\phi)$ is lifted by the conventional mechanism of supersymmetry breaking. In this case, since the sizes of the monopoles are comparable to the Hubble radius when they are formed, initially the wiggles cannot form coils. The coils are formed at much later period when $l_{\mathrm{wiggles}} \gg d$, which means $\epsilon_t \ll 1$.

As we have discussed above, a rough estimation of the characteristic properties of the networks of the brane necklaces is straightforward. Their evolution can be approximated by the evolution of the conventional $p \ll 1$ strings or the necklaces. On the other hand, one may suspect that the "vacua" on the strings might not be degenerated, and the brane necklaces might not behave as we have discussed above. We think this claim could be true. If the vacua on the strings are only quasi-degenerated, the tension of the strings in the different domains will be different by a small factor $\Delta\mu$. Due to $\Delta\mu \neq 0$, the monopoles may move along the strings, and may be annihilated or gathered to form coils on the strings. After the "true vacuum" dominates the necklaces, the reconnection probability of the necklaces will be enhanced up to $p \simeq 1$, because the other "false vacua" have shrunk to a point. Then the situation becomes quite the same as the evolution of the conventional $p \sim 1$ necklaces [28]. The only difference is that the monopoles are replaced by the coils, which can stabilize the loops of the strings.

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3. Conclusions and discussions

In this paper we have investigated the cosmological evolution of the cosmic strings in angled inflation. We have shown that the cosmic strings in angled inflation turn out to be the brane necklaces. The evolution of the networks of the brane necklaces is similar to the conventional $p \ll 1$ strings. However, the loops of the brane necklaces will shrink to produce superheavy winding states, which look like coils. In angled inflation, it is therefore possible to distinguish the brane necklaces from the conventional cosmic strings and necklaces. Moreover, the reconnection probability of the brane necklaces can be enhanced up to $p \sim 1$ if the degeneracy of the vacua is resolved by a small perturbation. We have made a rough estimation of the characteristic properties of the network of the brane necklaces. It is obvious that the numerical simulations are required to obtain more sensible results.

Here we should discuss what we can say about the conventional brane strings in the simplest scenario of $D\overline{D}$ brane inflation. As is already discussed in ref. [25, 26], it seems natural to think that the position of the daughter branes in the compactified space should be stabilized. On the other hand, however, there is no sensible reason that we have to believe that the potential has only a unique global minimum in the compactified space. Therefore, it is always important to consider daughter branes that have multiple minima in the compactified space. As a result, it is always natural to consider cosmic strings that have many domains on their world volume, which induces kinks interpolating between them. Even if the flat directions are stabilized at a unique minimum, winding states (coils) can appear if the Brownian motion induces windings around a nontrivial circle in the compactified space. Naively, the nontrivial circle means a conventional noncontractible circle that winds around compactified space. In generic cases, however, the nontrivial circle could be a valley of the potential that winds around a hill.

It is obvious that *any* branes that are produced by brane collision can have flat directions in the internal space. Then due to the Brownian motion, the cosmic strings will become the brane necklaces or the brane coils. The brane necklaces can therefore be produced in other generic brane models. If the necklaces are produced, they can be used to probe the internal structure of the compactified space. Moreover, future numerical simulations will provide us the important information about the undetermined parameters, which may put serious bounds on brane inflation.

Acknowledgments

We wish to thank K. Shima for encouragement, and our colleagues in Tokyo University for their kind hospitality.

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